WEEKLY TEST TYJ – 01 RAJPUR SOLUTION MATHEMATICS

31. (d) We have,

$\sin\theta + \operatorname{cosec}\theta = 2 \Rightarrow \sin^2\theta + 1 = 2\sin\theta$

$\Rightarrow \sin^2\theta - 2\sin\theta + 1 = 0$

$\Rightarrow (\sin\theta - 1)^2 = 0 \Rightarrow \sin\theta = 1$

Required value of $\sin^{10}\theta + \operatorname{cosec}^{10}\theta = (1)^{10} + \frac{1}{(1)^{10}} = 2$.

32. (b) Since $\operatorname{cosec}^2\theta = 1 + \cot^2\theta = 1 + \frac{9}{16} = \frac{25}{16}$

$\left(\because \tan\theta = -\frac{4}{3}\right)$

$\sin^2\theta = \frac{1}{\operatorname{cosec}^2\theta} = \frac{16}{25} \Rightarrow \sin\theta = \pm\frac{4}{5}$,

Both the values are acceptable, since $\tan\theta = -\frac{4}{3}$

i.e., θ lies in 2^{nd} or 4^{th} quadrant.

33. (b) $(\sin^2\theta + \cos^2\theta)^3 = (1)^3$

$\Rightarrow \sin^6\theta + \cos^6\theta + 3\sin^2\theta\cos^2\theta = 1$

and $\sin^4\theta + \cos^4\theta + 2\sin^2\theta\cos^2\theta = 1$

Both gives,

$2(\sin^6\theta + \cos^6\theta) - 3(\sin^4\theta + \cos^4\theta) + 1 = 0$.

34. (a) We know that one of the factor of the given expression is $\cos 90° = 0$.

Therefore $\cos 1°.\cos 2°.\cos 3°...\cos 179° = 0$.

35. (b) Given that $A + B = \frac{\pi}{4} \Rightarrow \tan(A+B) = \tan\frac{\pi}{4}$

$\Rightarrow \frac{\tan A + \tan B}{1 - \tan A\tan B} = 1$

$\Rightarrow \tan A + \tan B + \tan A\tan B = 1$

$\Rightarrow (1 + \tan A)(1 + \tan B) = 2$.

36. (c) $\tan x + \tan\left(\frac{\pi}{3} + x\right) + \tan\left(\frac{2\pi}{3} + x\right)$

$= \tan x + \frac{\tan x + \sqrt{3}}{1 - \sqrt{3}\tan x} + \frac{\tan x - \sqrt{3}}{1 + \sqrt{3}\tan x}$

$= \tan x + \frac{8\tan x}{1 - 3\tan^2 x} = \frac{3(3\tan x - \tan^3 x)}{1 - 3\tan^2 x} = 3\tan 3x$

Therefore, the given equation is $3\tan 3x = 3$

$\Rightarrow \tan 3x = 1$.

37. (c) $\tan 20^o \tan 40^o \tan 60^o \tan 80^o$

$= \frac{\sin 20^o \sin 40^o \sin 80^o \tan 60^o}{\cos 20^o \cos 40^o \cos 80^o}$

Here $N^r = (\sin 20^o \sin 40^o \sin 80^o)$

$= \frac{\sin 20^o}{2}(2\sin 40^o \sin 80^o)$

$= \frac{\sin 20^o}{2}(\cos 40^o - \cos 120^o)$

$= \frac{1}{2}\sin 20^o\left(1 - 2\sin^2 20^o + \frac{1}{2}\right)$

$$= \frac{1}{2}\sin 20^o\left(\frac{3}{2} - 2\sin^2 20^o\right) = \frac{\sin 60^o}{4} = \frac{\sqrt{3}}{8}$$

Now, we take $D' = \cos 20^o \cos 40^o \cos 80^o$

$$= \frac{\sin 2^3 20^o}{2^3 \sin 20^o} = \frac{\sin 160^o}{8\sin 20^o} = \frac{\sin 20^o}{8\sin 20^o} = \frac{1}{8}$$

$$\therefore \text{ Hence } \tan 20^o \tan 40^o \tan 80^o = \frac{\sqrt{3}/8}{1/8}$$

Therefore $\tan 20^o \tan 40^o \tan 60^o \tan 80^o = \sqrt{3}.\sqrt{3} = 3$.

38. (a) Since $\tan 3A = \dfrac{\tan A + \tan 2A}{1 - \tan A \tan 2A}$

$$\Rightarrow \tan 3A - \tan 2A - \tan A = \tan 3A \tan 2A \tan A.$$

39. (c) $\dfrac{\sin 3\theta + \sin 5\theta + \sin 7\theta + \sin 9\theta}{\cos 3\theta + \cos 5\theta + \cos 7\theta + \cos 9\theta}$

$$= \frac{(\sin 3\theta + \sin 9\theta) + (\sin 5\theta + \sin 7\theta)}{(\cos 3\theta + \cos 9\theta) + (\cos 5\theta + \cos 7\theta)}$$

$$= \frac{2\sin 6\theta \cos 3\theta + 2\sin 6\theta \cos\theta}{2\cos 6\theta \cos 3\theta + 2\cos 6\theta \cos\theta}$$

$$= \frac{2\sin 6\theta(\cos 3\theta + \cos\theta)}{2\cos 6\theta(\cos 3\theta + \cos\theta)} = \tan 6\theta.$$

40. (c) We have $b\sin\alpha = a\sin(\alpha + 2\beta) \Rightarrow \dfrac{a}{b} = \dfrac{\sin\alpha}{\sin(\alpha + 2\beta)}$

$$\Rightarrow \frac{a+b}{a-b} = \frac{\sin\alpha + \sin(\alpha + 2\beta)}{\sin\alpha - \sin(\alpha + 2\beta)} = \frac{2\sin(\alpha+\beta)\cos\beta}{-2\cos(\alpha+\beta)\sin\beta}$$

$$= -\tan(\alpha+\beta)\cot\beta = -\frac{\cot\beta}{\cot(\alpha+\beta)}.$$

41. (d) $\cos\dfrac{\pi}{7}.\cos\dfrac{2\pi}{7}.\cos\dfrac{4\pi}{7} = \left[\dfrac{\sin\left(2^3.\dfrac{\pi}{7}\right)}{2^3\sin\left(\dfrac{\pi}{7}\right)}\right] = \dfrac{\sin\dfrac{8\pi}{7}}{8\sin\dfrac{\pi}{7}} = -\dfrac{1}{8}.$

42. (b) $\dfrac{\tan 70^o - \tan 20^o}{\tan 50^o}$

$$= \frac{\dfrac{\sin 70^o}{\cos 70^o} - \dfrac{\sin 20^o}{\cos 20^o}}{\dfrac{\sin 50^o}{\cos 50^o}} = \frac{\dfrac{\sin 70^o\cos 20^o - \cos 70^o \sin 20^o}{\cos 70^o \cos 20^o}}{\dfrac{\sin 50^o}{\cos 50^o}}$$

$$= \frac{2}{2}\times\frac{\sin(70^o - 20^o)\cos 50^o}{\cos 70^o\cos 20^o\sin 50^o} = \frac{2\sin 50^o\cos 50^o}{2\cos 70^o\cos 20^o\sin 50^o}$$

$$= \frac{2\cos 50^o}{\cos 90^o + \cos 50^o} = \frac{2\cos 50^o}{0 + \cos 50^o} = 2.$$

43. (b) $\dfrac{\cot^2 15^o - 1}{\cot^2 15^o + 1} = \dfrac{\dfrac{\cos^2 15^o}{\sin^2 15^o} - 1}{\dfrac{\cos^2 15^o}{\sin^2 15^o} + 1}$

$$= \frac{\cos^2 15^o - \sin^2 15^o}{\cos^2 15^o + \sin^2 15^o} = \cos(30^o) = \frac{\sqrt{3}}{2}.$$

44. (a) $(\cos\alpha + \cos\beta)^2 + (\sin\alpha + \sin\beta)^2$

$$= \cos^2\alpha + \cos^2\beta + 2\cos\alpha\cos\beta + \sin^2\alpha + \sin^2\beta + 2\sin\alpha\sin\beta$$

$$= 2\{1 + \cos(\alpha - \beta)\} = 4\cos^2\left(\frac{\alpha - \beta}{2}\right).$$

45. (b) $\dfrac{\sqrt{1+\sin x} + \sqrt{1-\sin x}}{\sqrt{1+\sin x} - \sqrt{1-\sin x}} = \dfrac{\cos\dfrac{x}{2} + \sin\dfrac{x}{2} + \sin\dfrac{x}{2} - \cos\dfrac{x}{2}}{\cos\dfrac{x}{2} + \sin\dfrac{x}{2} - \sin\dfrac{x}{2} + \cos\dfrac{x}{2}}$

$$= \tan\frac{x}{2}.$$